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THE ROLE OF DAMPING IN SEISMIC ISOLATION

JAMES M. KELLY*
Earthquake Engineering Research Center, University of California, Berkeley, CA 94804, U.S.A.

SUMMARY

In the current code requirements for the design of base isolation systems for buildings located at near-fault sites, the design engineer is faced with very large design displacements for the isolators. To reduce these displacements, supplementary dampers are often prescribed. These dampers reduce displacements, but at the expense of significant increases in interstorey drifts and floor accelerations in the superstructure. An elementary analysis based on a simple model of an isolated structure is used to demonstrate this dilemma. The model is linear and is based on modal analysis, but includes the modal coupling terms caused by high levels of damping in the isolation system. The equations are solved by a method that avoids complex modal analysis. Estimates of the important response quantities are obtained by the response spectrum method. It is shown that as the damping in the isolation system increases, the contribution of the modal coupling terms due to isolator damping in response to the superstructure becomes the dominant term. The isolator displacement and structural base shear may be reduced, but the floor accelerations and interstorey drift are increased. The results show that the use of supplemental dampers in seismic isolation is a misplaced effort and alternative strategies to solve the problem are suggested. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: damping; floor accelerations; interstorey drift; isolation; near-fault sites

INTRODUCTION

In recent years the efficacy of seismic isolation technology has been under attack from an unexpected source. Several seismologists in California have published papers suggesting that base-isolated buildings are vulnerable to large pulse-like ground motions generated at near-fault locations.[1,2] Some evidence for pulse-type motion was observed in the Sylmar ground motion record in the 1994 Northridge, California, earthquake, which gave rise to some concern for the safety of base-isolated buildings. The concern of the seismologists that base-isolated buildings might perform poorly under such ground motions appears to rise from a misunderstanding of the dynamic behaviour of base-isolated structures, based on flawed models of isolation systems. Nevertheless, their predictions have influenced the design of base-isolated buildings in California.

As the required ground motions for buildings have increased in intensity, the isolation systems have increased in complexity, with the trend now toward very large isolators in combination with large viscous dampers. The first example of this new tendency is the San Bernardino County Medical Research Project.[3] The hospital has five base-isolated buildings, totalling 86,400 m^2.

* Correspondence to: James M. Kelly, Earthquake Engineering Research Center, National Information Service for Earthquake Engineering, Richmond, CA 94804, U.S.A. E-mail: jmkelly@eerc.berkeley.edu

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The total cost of the medical center is \$450 million, of which \$251 million is the construction cost. The isolation system combines 392 high-damping rubber isolators and 184 viscous dampers. The isolators are 500 mm in height and either 750 or 900 mm in diameter. The total cost of the isolators was \$5·1 million. The dampers, 3·7 m long and 360 mm in diameter, cost \$4·9 million. Each damper has a 1·2 m stroke and generates a force of 1·42 MN at a velocity of 1·5 m/s. The force in a damper is proportional to the velocity to the power 0·4.

The maximum displacements at the Maximum Capable Earthquake (MCE) for which the buildings were designed is 550 mm, but from an analysis of the final design[3] subjected to the Sylmar ground motion recorded in the 1994 Northridge earthquake, the computed displacements are less than half the design displacements (being 218 mm at centre-of-mass and 231 mm at the corners), indicating a substantial degree of conservatism in the design.

Other recent examples of this approach are the new Hayward City Hall that uses FPS isolators with a 3·0 s period and a set of viscous dampers, and the new Public Safety Building for the City of Berkeley for which 3·0 s period FPS isolators and viscous dampers are being proposed. The Kaiser Coronado Data Centre, which was built in southern California in 1989 as an isolated structure, has recently had its isolation system upgraded by the addition of viscous dampers.

Combining large viscous dampers with isolators underscores the extreme difficulty of getting the level of damping intrinsic to a hysteretic isolator system above 20 per cent equivalent viscous damping when the displacements become large. As a result, the isolation system designer who attempts to control the large code-mandated displacements through damping is forced to use supplemental dampers. Ironically, the dampers themselves, although controlling displacements, drive energy into higher modes and defeat the primary reason for using isolation—namely, the reduction of interstorey drift and floor acceleration. The basic concept of seismic isolation is that if the fundamental period of the fixed-based structure is much shorter than the isolated period, then the higher modes, which produce the floor accelerations and the interstorey drift, have very small participation factors.

There is also some question as to the effectiveness of an isolation system with as much damping used in the San Bernardino County Medical Centre. It is difficult to estimate the equivalent viscous damping in a case where there are so many non-linear elements having both velocity-dependent damping and hysteretic damping, but if the displacement response spectrum of the Sylmar record is computed for various levels of damping, then the result obtained by non-linear time-history analysis—namely 218 mm—is obtained at a nominal period of 3 s when the damping factor is 50 per cent. At this level of damping, participation of the higher modes must be important and the basic concept of isolation cannot hold.

CURRENT CODE REQUIREMENTS FOR SEISMICALLY ISOLATED STRUCTURES

The first building in the United States to use a seismic isolation system was completed in 1985. Although this building was publicized in national engineering magazines and visited by a great many engineers and architects from the United States and around the world, it was several years before construction of the second base-isolated building was begun. The acceptance of isolation as an anti-seismic design approach for some classes of buildings was clearly hampered in the United States by lack of a code covering base-isolated structures. To address this issue the Structural Engineers Association of Northern California (SEAONC) created a working group to develop design guidelines for isolated buildings.

The Seismology Committee of the Structural Engineers Association of California (SEAOC) is responsible for developing provisions for earthquake-resistant design of structures. These provisions, published as *Recommended Lateral Design Requirements and Commentary*[4] (generally referred to as the 'Blue Book'), have served as the basis for various editions of the Uniform Building Code (UBC), which is published by the International Conference of Building Officials (ICBO) and is the most widely used code for earthquake design. In 1986 the SEAONC sub-committee produced a document entitled *Tentative Seismic Isolation Design Requirements*[5] (known as the 'Yellow Book') as a supplement to the fourth edition of the Blue Book.

The approach and layout of the Yellow Book was intended to follow the Blue Book as closely as possible. Emphasis was placed on equivalent lateral force procedures, and, as in the Blue Book, the level of seismic input was that required for the design of fixed-base structures: a level of ground motion that has a 10 per cent change of being exceeded in a 50 year period. As in the Blue Book, dynamic methods of analysis were permitted, and for some types of structures required, but the simple statistically equivalent formulas provide a minimum level for the design.

This document, which included a useful commentary, was formulated around the basic theory of seismic isolation. The fixed-based frequency had to be greater than three times the isolation frequency. The stresses in the superstructure were to be computed from a uniform distribution of shear, and the structural reduction factors were one-half of those for conventional structures, thus ensuring very little inelastic behaviour. Only one level of design earthquake was used, but it was required that the isolators be tested at a displacement of 1·25 times the design displacement. Thus, no MCE design was required, but the bearings had to be checked for a defacto MCE displacement. In all later versions of the seismic isolation codes, MCE ground motions are specified, and, once these are manifest, it is difficult for the design engineer not to base the design on them.

The SEAOC Seismology Committee formed a subcommittee in 1988 to produce an isolation design document entitled *General Requirements for the Design and Construction of Seismic-Isolated Structures*.[6] In 1990 this was published as an appendix to the fifth edition of the Blue Book and later adopted by ICBO as an appendix to the seismic provisions in the 1991 UBC.[7] This version of the code includes the static method of analysis and retains a minimum level of design based on a factor of the static analysis values, but increases the number of situations where dynamic analysis is mandatory.

Another code document, developed for the design of base-isolated hospitals in California, has been adopted by the Building Safety Board (BSB) of the Office of State Architect. Entitled *An Acceptable Method for Design and Review of Hospital Buildings Utilizing Base Isolation*,[8] these guidelines were developed in part by SEAONC for the BSB and are similar to both the SEAONC requirements and the UBC code. The version adopted by the BSB in 1989 was revised in January 1992 and includes additional requirements.

The UBC codes and the BSB codes differ from the early SEAONC guidelines in that they explicitly require that the design must be based on two levels of seismic input. A Design Basis Earthquake (DBE) is defined as the level of earthquake ground shaking that has a 10 per cent probability of being exceeded in a 50 year period. The design provisions for this level of input require that the structure above the isolation system remains essentially elastic. The second level of input is defined as the Maximum Capable Earthquake (MCE), which is the maximum level of earthquake ground shaking that may be expected at the site within the known geological framework. This is taken as that earthquake ground motion that has a 10 per cent probability of being exceeded in 250 years. The isolation system should be designed and tested for this level of

seismic input, and all building separations and utilities that cross the isolation interface should be designed to accommodate the forces and displacements for this level of seismic input.

Changes incorporated into the 1994 UBC[9] regulations for isolated buildings made them even more conservative in some aspects than the 1991 version. The 1994 regulations restricted further the use of static analysis, although the code continued to require static analysis in all cases in order to provide various minimum levels below which design values obtained by dynamic analysis cannot fall. The design had to be based on two levels of earthquake input: the DBE, which is used to calculate the total design displacement of the isolation system and the forces in the superstructure, and the MCE, which is used to calculate what is referred to as the total maximum displacement of the isolation system for which the system must be shown to be safe. The vertical distribution of force was changed from a uniform one to a triangular one that is generally used for fixed-base structures. The superstructure was to be designed for forces produced by the isolation system at the design displacement reduced by certain reduction factors, which were now less than the previous factors (generally one-half of those for fixed-base structures). These two changes for the design forces resulted in ensuring that the superstructure will be elastic at the DBE.

The recently published 1997 UBC[10] is very different from the 1994 version in lay-out. Much of the simplicity of the earlier code has been lost since the base isolation section is no longer self-contained. In addition to being more complex, it is also more conservative.

Suppose we have a building located 2 km from a known active fault of seismic source type A (defined in Table 16-U[10]). Assume that the soil type is stiff soil, S_{D} (defined in Table 16-J[10]). As an example, let us assume that the period of the isolated building is 2.5 s at the DBE and the MCE, and assume 15 per cent damping.

The static formulas for DBE displacement D_{D} and MCE displacement D_{M} are

$$D_{\mathrm{D}} = \frac{g}{4\pi^2}\frac{C_{\mathrm{VD}}T_{\mathrm{D}}}{B_{\mathrm{D}}}$$

$$D_{\mathrm{M}} = \frac{g}{4\pi^2}\frac{C_{\mathrm{VM}}T_{\mathrm{M}}}{B_{\mathrm{M}}}$$

The quantity C_{VD} is the same as C_{V} in Table 16-R[10] and for $Z = 0{\cdot}4$ and S_{D} is given as

$$C_{\mathrm{VD}} = 0{\cdot}64 N_{\mathrm{V}}$$

The quantity N_{V} is a near-fault factor given in Table 16-T[10] and for source type A at 2 km distance is 2·0. With these numbers and $T_{\mathrm{D}} = 2{\cdot}5$ s, $B_{\mathrm{D}} = 1{\cdot}35$, we have

$$D_{\mathrm{D}} = 58{\cdot}9 \text{ cm}$$

The quantity C_{VM} is given in Table A-16-G[10] as $1{\cdot}6 M_{\mathrm{M}} Z N_{\mathrm{V}}$, where 1·6 is for the soil factor and M_{M} is the MCE Response Coefficient; for $ZN_{\mathrm{V}} \geqslant 0{\cdot}50$, it is 1·2 (from Table A-16-D[10]). With all of these numbers we have

$$D_{\mathrm{VM}} = 70{\cdot}6 \text{ cm}$$

Further multipliers for torsion need to be applied to both of these results. The minimum multiplier for torsion is 1·1, meaning that the designer is faced with a DBE design displacement of at least 64·8 cm and an MCE displacement of 77·7 cm.

To compare these numbers with the 1994 UBC requirements[9] for the same site, soil, period, and damping, we have the following result: the design displacement is given by

$$D = \frac{10NZS_{\mathrm{I}}T_{\mathrm{I}}}{B}$$

where $S_{\mathrm{I}} = 1{\cdot}4$ and $N = 1{\cdot}25$; $T_{\mathrm{I}} = 2{\cdot}5$ and $B = 1{\cdot}35$ are as before. We find

$$D = 34{\cdot}3 \text{ cm}$$

The default allowance for torsion gives

$$D_{\mathrm{T}} = 37{\cdot}8 \text{ cm}$$

and the MCE factor, $M_{\mathrm{M}} = 1{\cdot}25$, gives $D_{\mathrm{TM}} = 42{\cdot}2$ cm. It is worth noting that if the site is at a distance greater than 15 km from the fault, the differences in the displacement quantities are much less; the greater conservatism is for near-fault sites.

It is the MCE displacement that is the critical parameter for designing the isolator. Although some further reduction for higher damping may be possible (Table A-16-D[10]), it is very difficult to get much damping in hysteretic isolators at large displacements because the approximate formula that translates the hysteretic damping (whereby the energy dissipated in a cycle is proportional to displacement) into equivalent viscous damping (whereby the energy dissipated is proportional to displacement squared) leads to an effective damping that is inversely proportional to displacement.

Of course, the values of displacement obtained by the code formulas cannot be taken as design values for a structure located 2 km from a known fault. The code mandates that all structures located less than 10 km from an active fault must be designed using dynamic analysis, and this must be done using a site-specific design spectra. If a set of time-history ground motions are used for design, they must be scaled to be compatible with the site-specific spectrum by a code procedure that imposes a 30 per cent increase in the displacements.

The specified procedure for the generation of spectrum-matched time histories by its very nature represents an increased seismic input to the structure. The static formula is based on a constant velocity spectrum that is consistent over the isolation system period range, with the acceleration spectrum designated in Figure 16-U of the code.[10] Suppose that the site-specific spectrum is, in fact, that spectrum. The rule is that the selected earthquake time histories are linearly scaled by factors that ensure that the average of the Square Root of Sum of Squares (SRSS) of the spectral values over the isolation range should not fall below 1·3 times the target spectra by more than 10 per cent. Suppose now that these time histories are applied to a 5 per cent damped linear isolation system. The earthquakes will produce displacements in two orthogonal directions, and the maximum displacement will be the SRSS of the displacements in the two directions. When these are averaged over the entire range of records, we are back to 1·3 times the displacement in the target spectrum. The static formula is unrelated to direction, the maximum can be any direction. Thus, the procedure imposes a 30 per cent increase in ground motion. Taken together, the several steps required for dynamic design make it almost inevitable that the displacements from the static formula will be exceeded.

Because MCE displacements of the order of 76·2 cm will lead to very large isolators, costly flexible connections for utilities, and an extensive and expensive loss of space for a seismic gap or moat, the isolation designer, in an attempt to reduce the large design displacements, incorporates mechanical dampers.

THEORETICAL BASIS OF SEISMIC ISOLATION

The linear theory of seismic isolation is given in detail by Kelly.[11] A concise outline of the analysis will be given in this section. The theory is based on a two mass structural model, as shown in Figure 1. The mass, m, is intended to represent the superstructure of the building and m_b the mass of the base floor above the isolation system. The structure stiffness and damping are represented by k_s, c_s, and the stiffness and damping of the isolation by k_b, c_b. Absolute displacements of the two masses are denoted by u_s and u_b, but it is convenient to use relative displacemnts, and accordingly define

$$v_b = u_b - u_g$$

$$v_s = u_s - u_b$$

where u_g is the ground displacement. This choice of relative displacements is particularly convenient for this analysis because the two important results will be the isolation system displacement, represented here by v_b, and the interstorey drift, represented by v_s.

In terms of these quantities, the basic equations of motion of the two-degree-of-freedom model are

$$(m + m_b)\ddot{v}_b + m\ddot{v}_s + c_b\dot{v}_b + k_b v_b = -(m + m_b)\ddot{u}_g \tag{1}$$

$$m\ddot{v}_b + m\ddot{v}_s + c_s\dot{v}_s + k_s v_b = -m\ddot{u}_g \tag{2}$$

which can be written in matrix notation as

$$\begin{bmatrix} M & m \\ m & m \end{bmatrix}\begin{Bmatrix} \ddot{v}_b \\ \ddot{v}_s \end{Bmatrix}\begin{bmatrix} c_b & 0 \\ 0 & c_s \end{bmatrix}\begin{Bmatrix} \dot{v}_b \\ \dot{v}_s \end{Bmatrix} + \begin{bmatrix} k_b & 0 \\ 0 & k_s \end{bmatrix}\begin{Bmatrix} v_b \\ v_s \end{Bmatrix} = \begin{bmatrix} M & m \\ m & m \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\ddot{u}_g \tag{3}$$

where $M = m + m_b$, i.e. in matrix notation

$$\mathbf{M}\ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}} + \mathbf{K}\mathbf{v} = -\mathbf{M}\mathbf{r}\ddot{u}_g$$

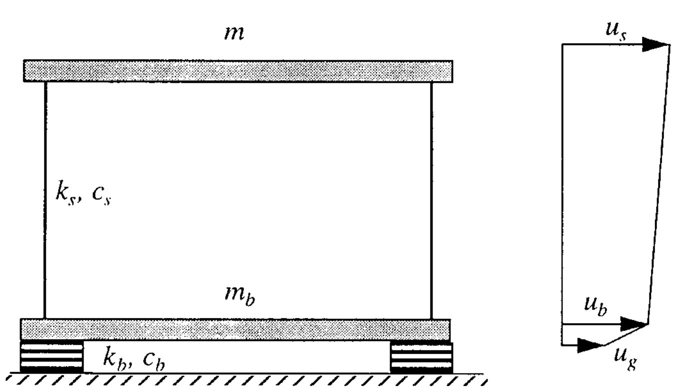

Figure 1. Parameters of 2DOF isolated system[12]

We define a mass ratio γ as

$$\gamma = \frac{m}{m + m_b} = \frac{m}{M} \tag{4}$$

and the nominal frequencies ω_b and ω_s given by

$$\omega_b^2 = \frac{k_b}{m + m_b}$$
$$\omega_s^2 = \frac{k_s}{m} \tag{5}$$

and damping factors β_b and β_s given by

$$2\omega_b \beta_b = \frac{c_b}{m + m_b}$$
$$2\omega_s \beta_s = \frac{c_s}{m} \tag{6}$$

and assume that $\omega_b^2/\omega_s^2 = \varepsilon$ and that $\varepsilon = 0(10^{-2})$.

The classical modes of the combined system will be denoted by $\underline{\phi}^1$ and $\underline{\phi}^2$, where

$$\underline{\phi}^{i^{\mathrm{T}}} = (\phi_b^i, \phi_s^i) \qquad i = 1, 2$$

with frequencies ω_1 and ω_2. The characteristic equation for the frequencies is

$$(1 - \gamma)\omega^4 - (\omega_s^2 + \omega_b^2)\omega^2 + \omega_b^2 \omega_s^2 = 0 \tag{7}$$

the solutions of which are

$$\omega_1^2 = \frac{1}{2(1-\gamma)} \{\omega_b^2 + \omega_s^2 - [(\omega_b^2 - \omega_s^2)^2 + 4\gamma\omega_b^2\omega_s^2]^{1/2}\}$$
$$\omega_2^2 = \frac{1}{2(1-\gamma)} \{\omega_b^2 + \omega_s^2 + [(\omega_b^2 - \omega_s^2)^2 + 4\gamma\omega_b^2\omega_s^2]^{1/2}\} \tag{8}$$

and to first order in ε are given by

$$\omega_1^2 = \omega_b^2(1 - \gamma\varepsilon)$$
$$\omega_2^2 = \frac{\omega_s^2}{1-\gamma}(1 + \gamma\varepsilon) \tag{9}$$

and the mode shapes with ($\phi_b^i = 1$), $i = 1, 2$, are

$$\underline{\phi}^{1^{\mathrm{T}}} = (1, \varepsilon) \qquad \underline{\phi}^{2^{\mathrm{T}}} = \left\{1, -\frac{1}{\gamma}[1 - (1-\gamma)\varepsilon]\right\} \tag{10}$$

To express the original displacements in modal co-ordinates, we write

$$v_{\mathrm{b}} = q_1 \phi_{\mathrm{b}}^1 + q_2 \phi_{\mathrm{b}}^2$$

$$v_{\mathrm{s}} = q_1 \phi_{\mathrm{s}}^1 + q_2 \phi_{\mathrm{s}}^2$$

where q_1, q_2 are time-dependent modal coefficients.

We note that modal quantities M_i, L_i are given by

$$M_i = \underset{\sim}{\phi}^{i^{\mathrm{T}}} \mathbf{M} \underset{\sim}{\phi}^i$$

$$M_i L_i = \underset{\sim}{\phi}^{i^{\mathrm{T}}} \mathbf{Mr}$$

To first order in ε, these are

$$\begin{aligned} M_1 &= M(1 - 2\gamma\varepsilon) \\ M_2 &= M \frac{(1-\gamma)[1 - 2(1-\gamma)\varepsilon]}{\gamma} \end{aligned} \tag{11}$$

and

$$\begin{aligned} L_1 &= 1 - \gamma\varepsilon \\ L_2 &= \gamma\varepsilon \end{aligned} \tag{12}$$

When v_{b}, v_{s} in equations (1) and (2) are expressed in terms of $\underset{\sim}{\phi}^1$ and $\underset{\sim}{\phi}^2$, we have two equations in the modal coefficients (q_1, q_2) of the form

$$\ddot{q}_1 + 2\omega_1\beta_1\dot{q}_1 + \lambda_1\dot{q}_2 + \omega_1^2 q_1 = -L_1\ddot{u}_{\mathrm{g}} \tag{13}$$

$$\ddot{q}_2 + \lambda_2\dot{q}_1 + 2\omega_2\beta_2\dot{q}_2 + \omega_2^2 q_2 = -L_2\ddot{u}_{\mathrm{g}} \tag{14}$$

The terms $2\omega_1\beta_1$ and $2\omega_2\beta_2$ are computed from

$$M_i 2\omega_i\beta_i = \underset{\sim}{\phi}^{i^{\mathrm{T}}} \begin{bmatrix} c_{\mathrm{b}} & 0 \\ 0 & c_{\mathrm{s}} \end{bmatrix} \underset{\sim}{\phi}^i$$

from which we obtain

$$2\omega_1\beta_1 = 2\omega_{\mathrm{b}}\beta_{\mathrm{b}}(1 - 2\gamma\varepsilon)$$

and

$$2\omega_2\beta_2 = \frac{1}{1-\gamma}(2\omega_{\mathrm{s}}\beta_{\mathrm{s}} + 2\gamma\omega_{\mathrm{b}}\beta_{\mathrm{b}})$$

leading to

$$\beta_1 = \beta_{\mathrm{b}}\left(1 - \frac{3}{2}\gamma\varepsilon\right) \tag{15}$$

$$\beta_2 = \frac{\beta_{\mathrm{s}} + \gamma\beta_{\mathrm{b}}\varepsilon^{1/2}}{(1-\gamma)^{1/2}}\left(1 - \frac{\gamma\varepsilon}{2}\right) \tag{16}$$

The coupling coefficients λ_1 and λ_2 are computed from

$$\lambda_1 M_1 = \underset{\sim}{\phi}^{1^{\mathrm{T}}} \begin{bmatrix} c_{\mathrm{b}} & 0 \\ 0 & c_{\mathrm{s}} \end{bmatrix} \underset{\sim}{\phi}^2$$

and

$$\lambda_2 M_2 = \underset{\sim}{\phi}^{2^{\mathrm{T}}} \begin{bmatrix} c_{\mathrm{b}} & 0 \\ 0 & c_{\mathrm{s}} \end{bmatrix} \underset{\sim}{\phi}^1 = \lambda_1 M_1$$

Thus

$$\lambda_1 M_1 = (1, \varepsilon) \begin{bmatrix} c_{\mathrm{b}} & 0 \\ 0 & c_{\mathrm{s}} \end{bmatrix} \begin{pmatrix} 1 \\ -a \end{pmatrix} = c_{\mathrm{b}} - \varepsilon a c_{\mathrm{s}}, \qquad a = \frac{1}{\gamma}[1 - (1 - \gamma)\varepsilon]$$

Using (M_1, M_2) from equation (11), we have

$$\begin{aligned} \lambda_1 &= \frac{2\omega_{\mathrm{b}}\beta_{\mathrm{b}}M - \varepsilon\{1/\gamma[1-(1-\gamma)\varepsilon]\}2\omega_{\mathrm{s}}\beta_{\mathrm{s}}m}{M(1-2\gamma\varepsilon)} \\ &= 2\omega_{\mathrm{b}}\beta_{\mathrm{b}}(1-2\gamma\varepsilon) - 2\omega_{\mathrm{s}}\beta_{\mathrm{s}}(1-2\gamma\varepsilon)\varepsilon \\ &= 2\omega_{\mathrm{b}}[\beta_{\mathrm{b}}(1-2\gamma\varepsilon) - \varepsilon^{1/2}\beta_{\mathrm{s}}] \end{aligned} \tag{17}$$

and

$$\begin{aligned} \lambda_2 &= \frac{2\omega_{\mathrm{b}}\beta_{\mathrm{b}}M - \varepsilon\{1/\gamma[1-(1-\gamma)\varepsilon]\}2\omega_{\mathrm{s}}\beta_{\mathrm{s}}m}{[M(1-\gamma)]/\gamma[1-2(1-\gamma)\varepsilon]} \\ &= (2\omega_{\mathrm{b}}\beta_{\mathrm{b}} - \varepsilon 2\omega_{\mathrm{s}}\beta_{\mathrm{s}})[1+2(1-\gamma)\varepsilon]\frac{\gamma}{1-\gamma} \\ &= 2\omega_{\mathrm{b}}\left\{\beta_{\mathrm{b}}[1+2(1-\gamma)\varepsilon] - \varepsilon^{1/2}\beta_{\mathrm{s}}\right\}\frac{\gamma}{1-\gamma} \end{aligned} \tag{18}$$

ANALYSIS OF COUPLED DYNAMIC EQUATIONS

In most structural applications it is assumed that the damping is small enough that the effect of the off-diagonal components (designated in this paper as λ_1 and λ_2) are negligible and that the required solution can be obtained from the uncoupled modal equations of motions. In the case of seismic isolation, the neglect of the off-diagonal components leads to very simple results for base displacement, base shear, and interstorey drift,[11] and these simple results formed the basis of the earlier design approaches as exemplified by the 1986 SEAONC Yellow Book.[5]

In many isolated structures designed according to the most recent California design codes, the code requirements are so conservative that the designers are using additional viscous dampers in an attempt to control the large design displacements, and damping factors for the isolation system of the order of 0·50 are obtained. Clearly, at this level of damping the equations cannot remain uncoupled and a complex modal analysis should be used. In complex modal analysis, however, we lose the physical insight that led to the simple results of the uncoupled solution.[11] For this reason a similar approximation to that employed there will be used in this section to

demonstrate the effect of high levels of damping in the isolation system on the response of the structure.

It is interesting to note that to zero order in ε, the four damping terms are

$$2\omega_1\beta_1 = 2\omega_b\beta_b$$
$$2\omega_2\beta_2 = \frac{1}{1-\gamma}2\omega_s\beta_s$$
$$\lambda_1 = 2\omega_b\beta_b$$
$$\lambda_2 = 2\omega_b\beta_b\frac{\gamma}{1-\gamma}$$

so that the off-diagonal components are of the same order as the diagonal terms. Recalling that $L_1 \approx 0(1)$ and $L_2 \approx 0(\varepsilon)$, we assume that the influence of $\lambda_1\dot{q}_2$ on the result for q_1 is negligible, but the influence of $\lambda_2\dot{q}_1$ on q_2 could be significant. Thus, we assume that equations (13) and (14) are modified, so that $q_1(t)$ is given by the solution of

$$\ddot{q}_1 + 2\omega_1\beta_1\dot{q}_1 + \omega_1^2 q_1 = -L_1\ddot{u}_g$$

and $q_2(t)$ by

$$\ddot{q}_2(t) + 2\omega_2\beta_2\dot{q}_2 + \omega_2^2 q_2 = -L_2\ddot{u}_g - \lambda_2\dot{q}_1$$

To aid in simplifying the solution, it is useful to take the Laplace Transform of these equations using

$$\text{L.T.}\,[f(t)] = \int_0^\infty e^{-st}f(t)\,dt = \bar{f}(s)$$

In terms of the Laplace Transform, we have

$$\bar{q}_1(s) = -\frac{L_1\bar{a}(s)}{s^2 + 2\omega_1\beta_1 s + \omega_1^2}$$
$$\bar{q}_2(s) = -\frac{L_2\bar{a}(s)}{s^2 + 2\omega_2\beta_2 s + \omega_2^2} + \frac{\lambda_2 L_1 s\bar{a}(s)}{(s^2 + 2\omega_2\beta_2 s + \omega_2^2)(s^2 + 2\omega_1\beta_1 s + \omega_1^2)}$$
$$= -L_2 A_1(s)\bar{a}(s) + \lambda_2 L_1 A_2(s)\bar{a}(s)$$

where $\bar{a}(s) = \text{L.T.}[\ddot{u}_g]$. The term $A_2(s)$ can be reduced by partial fractions to

$$A_2(s) = \frac{a + bs}{(s^2 + 2\omega_1\beta_1 s + \omega_1^2)} + \frac{c + ds}{(s^2 + 2\omega_2\beta_2 s + \omega_2^2)}$$

where after considerable manipulation we find

$$\begin{aligned} a &= \omega_1^2(2\omega_2\beta_2 - 2\omega_1\beta_1)/D \\ b &= (\omega_2^2 - \omega_1^2)/D \\ c &= -\omega_2^2(2\omega_2\beta_2 - 2\omega_1\beta_1)/D \\ d &= -(\omega_2^2 - \omega_1^2)/D \end{aligned} \tag{19}$$

and

$$D = (\omega_2^2 - \omega_1^2)^2 + (2\omega_2\beta_2 - 2\omega_1\beta_1)(\omega_1^2 2\omega_2\beta_2 - \omega_2^2 2\omega_1\beta_1) \tag{20}$$

The inversion of the two terms of $A_2(s)$ follows from

$$\text{L.T.}^{-1}\left[\frac{(s-\alpha)}{(s-\alpha)^2+\gamma^2}\right] = \mathrm{e}^{\alpha t}\cos\gamma t$$

and

$$\text{L.T.}^{-1}\left[\frac{1}{(s-\alpha)^2+\gamma^2}\right] = \frac{1}{\gamma}\mathrm{e}^{\alpha t}\sin\gamma t$$

so that the inversion of

$$\frac{a+bs}{(s^2+2\omega_1\beta_1 s+\omega_1^2)}$$

is

$$b\mathrm{e}^{-\omega_1\beta_1 t}\cos\bar{\omega}_1 t + (a - b\bar{\omega}_1\beta_1)\frac{\mathrm{e}^{-\omega_1\beta_1 t}\sin\bar{\omega}_1 t}{\bar{\omega}_1}$$

and of

$$\frac{c+ds}{(s^2+2\omega_2\beta_2 s+\omega_2^2)}$$

is

$$d\mathrm{e}^{-\omega_2\beta_2 t}\cos\bar{\omega}_2 t + (c - d\omega_2\beta_2)\frac{\mathrm{e}^{-\omega_2\beta_2 t}\sin\bar{\omega}_2 t}{\omega_2}$$

where

$$\bar{\omega}_1 = \omega_1(1-\beta_1^2)^{1/2} \qquad \bar{\omega}_2 = \omega_2(1-\beta_2^2)^{1/2}$$

The final result of $q_1(t)$ and $q_2(t)$ is obtained by convolution and substitution of a, b, c, and d from equation (19) as

$$q_1(t) = -\frac{L_1}{\bar{\omega}_1}\int_0^t \ddot{u}_{\mathrm{g}}(t-\tau)\mathrm{e}^{-\omega_1\beta_1\tau}\sin\bar{\omega}_1\tau\,\mathrm{d}\tau \tag{21}$$

$$\begin{aligned} q_2(t) = &- L_2\frac{1}{\bar{\omega}_2}\int_0^t \mathrm{e}^{-\omega_2\beta_2(t-\tau)}\sin\bar{\omega}_2(t-\tau)\ddot{u}_{\mathrm{g}}(\tau)\,\mathrm{d}\tau \\ &+ \lambda_2 L_1\left\{\frac{\omega_2^2-\omega_1^2}{D}\int_0^t \mathrm{e}^{-\omega_1\beta_1(t-\tau)}\cos\bar{\omega}_1(t-\tau)\ddot{u}_{\mathrm{g}}(\tau)\,\mathrm{d}\tau\right. \\ &+ \frac{\omega_1^2(2\omega_2\beta_2)-(\omega_1^2+\omega_2^2)\omega_1\beta_1}{D}\frac{1}{\bar{\omega}_1}\int_0^t \mathrm{e}^{-\omega_1\beta_1(t-\tau)}\sin\bar{\omega}_1(t-\tau)\ddot{u}_{\mathrm{g}}(\tau)\,\mathrm{d}\tau \end{aligned}$$

$$- \frac{\omega_2^2 - \omega_1^2}{D} \int_0^t \mathrm{e}^{-\omega_2\beta_2(t-\tau)} \cos \bar{\omega}_2(t-\tau)\ddot{u}_{\mathrm{g}}(\tau)\,\mathrm{d}\tau$$
$$\left. - \frac{(\omega_1^2 + \omega_2^2)\omega_2\beta_2 - \omega_2^2 2\omega_1\beta_1}{D} \frac{1}{\bar{\omega}_2} \int_0^t \mathrm{e}^{-\omega_2\beta_2(t-\tau)} \sin \bar{\omega}_2(t-\tau)\ddot{u}_{\mathrm{g}}(\tau)\,\mathrm{d}\tau \right\} \tag{22}$$

These convolution integrals can be computed for any choice of $\ddot{u}_{\mathrm{g}}(t)$, but for the purpose of this demonstration it is necessary only to have a sense of the order of magnitude of the results.

Terms in ω_1, ω_2 can be expressed in terms of the nominal frequencies ω_{b}, ω_{s} by the use of equation (8), from which we have

$$\omega_2^2 - \omega_1^2 = \frac{1}{1-\gamma}[(\omega_{\mathrm{s}}^2 - \omega_{\mathrm{b}}^2)^2 - 4\gamma\omega_{\mathrm{b}}^2\omega_{\mathrm{s}}^2]^{1/2}$$
$$\omega_2^2 + \omega_1^2 = \frac{\omega_{\mathrm{s}}^2 + \omega_{\mathrm{b}}^2}{1-\gamma}$$
$$\omega_1^2\omega_2^2 = \frac{\omega_{\mathrm{s}}^2\omega_{\mathrm{b}}^2}{1-\gamma}$$

The denominator D of each term in equation (22) can be written as

$$D = (\omega_2^2 - \omega_1^2)^2 - 4\omega_1^2\omega_2^2(\beta_1^2 + \beta_2^2) + 4\omega_1\omega_2(\omega_1^2 + \omega_2^2)\beta_1\beta_2$$

which reduces to

$$D = \frac{1}{(1-\gamma)^2}\Big\{(\omega_{\mathrm{s}}^2 - \omega_{\mathrm{b}}^2)^2 + 4\gamma\omega_{\mathrm{s}}^2\omega_{\mathrm{b}}^2 - 4(1-\gamma)\omega_{\mathrm{s}}^2\omega_{\mathrm{b}}^2\beta_1^2\beta_2^2$$
$$+ 4(1-\gamma)^{1/2}\omega_{\mathrm{s}}\omega_{\mathrm{b}}(\omega_{\mathrm{s}}^2 + \omega_{\mathrm{b}}^2)\beta_1\beta_2\Big\} \tag{23}$$

A further reduction of each term is possible if we assume the following orders of magnitude:

$$\gamma = 0(1) \qquad \frac{\omega_{\mathrm{b}}^2}{\omega_{\mathrm{s}}^2} = \varepsilon \ll 1$$
$$\beta_1^2, \beta_2^2, \beta_1\beta_2 = 0(\varepsilon)$$

To the first order in ε, we have

$$D = \frac{\omega_{\mathrm{s}}^4}{(1-\gamma)^2}[1 - 2(1-2\gamma)\varepsilon]$$

and the multipliers of each integral become

$$\frac{\omega_2^2 - \omega_1^2}{D} = \frac{\omega_{\mathrm{s}}^2}{1-\gamma}[1 - (1-2\gamma)\varepsilon]\frac{1}{D} = \frac{(1-\gamma)}{\omega_{\mathrm{s}}^2}[1 + (1-2\gamma)\varepsilon]$$
$$\frac{\omega_1^2(2\omega_2\beta_2) - (\omega_1^2 - \omega_2^2)\omega_1\beta_1}{D\omega_1} = -\frac{(1-\gamma)}{\omega_{\mathrm{s}}^2}\beta_1$$
$$\frac{(\omega_1^2 + \omega_2^2)\omega_2\beta_2 - \omega_2^2 2\omega_1\beta_1}{D\omega_2} = \frac{1-\gamma}{\omega_{\mathrm{s}}^2}\beta_2$$

giving for the four terms in parenthesis in equation (22)

$$\frac{1-\gamma}{\omega_s^2}\left\{[1+(1-2\gamma)\varepsilon]\left[\int_0^t e^{-\omega_1\beta_1(t-\tau)}\cos\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau-\int_0^t e^{-\omega_2\beta_2(t-\tau)}\cos\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau\right]\right.$$

$$\left.-\beta_1\int_0^t e^{-\omega_1\beta_1(t-\tau)}\sin\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau+\beta_2\int_0^t e^{-\omega_2\beta_2(t-\tau)}\sin\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau\right\}$$

The results for q_1 and q_2 to first order in ε are thus

$$q_1(t)=-\frac{L_1}{\bar{\omega}_1}\int_0^t e^{-\omega_1\beta_1(t-\tau)}\sin\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau \tag{24}$$

$$\begin{aligned}q_2(t)=&-\frac{L_2}{\bar{\omega}_2}\int_0^t e^{-\omega_2\beta_2(t-\tau)}\sin\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau\\&+\lambda_2L_1\frac{1-\gamma}{\omega_s^2}\left\{[1+(1-2\gamma\varepsilon)]\left[\int_0^t e^{-\omega_1\beta_1(t-\tau)}\cos\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau\right.\right.\\&\left.-\int_0^t e^{-\omega_2\beta_2(t-\tau)}\cos\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau\right]\\&-\beta_1\int_0^t e^{-\omega_1\beta_1(t-\tau)}\sin\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau\\&\left.+\beta_2\int_0^t e^{-\omega_2\beta_2(t-\tau)}\sin\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau\right\}\end{aligned} \tag{25}$$

It is convenient to denote the convolution intergrals in equations (24) and (25) by I_1, I_2, I_3, and I_4, where

$$I_1=\int_0^t e^{-\omega_1\beta_1(t-\tau)}\sin\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau$$

$$I_2=\int_0^t e^{-\omega_2\beta_2(t-\tau)}\sin\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau$$

$$I_3=\int_0^t e^{-\omega_1\beta_1(t-\tau)}\cos\bar{\omega}_1(t-\tau)\ddot{u}_g(\tau)\,d\tau$$

$$I_4=\int_0^t e^{-\omega_2\beta_2(t-\tau)}\cos\bar{\omega}_2(t-\tau)\ddot{u}_g(\tau)\,d\tau$$

In this analysis the quantities of interest are the interstorey drift and the floor accelerations, which are represented in this simple model by v_s and $\ddot{u}_s$. In this simple model they are related by

$$|\ddot{u}_s|_{max}=k_s|v_s|_{max}/m$$

so that the evaluation of v_s will also provide the floor acceleration. The interstorey drift, v_s, is given by

$$v_s = q_1 \phi_s^1 + q_2 \phi_s^2$$

leading to

$$\begin{aligned} v_s = &- \varepsilon \frac{L_1}{\omega_1} I_1 + \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\frac{L_2}{\omega_2} I_2 \\ &- \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\lambda_2 L_1 \frac{1-\gamma}{\omega_s^2}\left\{[1+(1-2\gamma\varepsilon)](I_3 - I_4) - \beta_1 I_1 + \beta_2 I_2\right\} \end{aligned} \tag{26}$$

It is useful here to seperate the three contributions to the drift as follows:

(i) that produced by the base shear generated by the isolation system

$$v_s^{(1)} = - \varepsilon \frac{L_1}{\omega_1} I_1 \tag{27}$$

(ii) that from the uncoupled modal equations

$$v_s^{(2)} = \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\frac{L_2}{\omega_2} I_2 \tag{28}$$

and
(iii) that from the coupling terms, which is generally neglected in most analyses

$$v_s^{(3)} = - \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\lambda_2 L_2 \frac{1-\gamma}{\omega_s^2}\left\{[1+(1-2\gamma\varepsilon)](I_3 - I_4) - \beta_1 I_1 + \beta_2 I_2\right\} \tag{29}$$

The convolution integrals, I_1, I_2, I_3, and I_4, can be estimated for the purpose of this demonstrative analysis by response spectrum methods. We recognize that

$$\frac{1}{\omega_1}|I_1|_{\max} = S_D(\omega_1, \beta_1)$$

$$\frac{1}{\omega_2}|I_2|_{\max} = S_D(\omega_2, \beta_2)$$

where S_D is the displacement response spectrum.

The expression

$$\left| \int_0^t e^{-\omega\beta(t-\tau)} \cos\bar{\omega}(t-\tau)\ddot{u}_g(\tau)\,d\tau \right|_{\max}$$

is the relative velocity response spectrum, $S_{RV}(\omega, \beta)$, for a single-degree-of-freedom oscillator of frequency ω and damping factor β. This we approximate by the pseudo-velocity response spectrum, $S_V(\omega, \beta)$, given by $\omega S_D(\omega, \beta)$. The peak values of the four convolution integrals in parenthesis in equation (25) will occur at different times and should be added by the SRSS method, leading to estimates of the maxima of the three contributions to v_s.

We have

$$|v_s^{(1)}|_{\max} = \varepsilon L_1 S_D(\omega_1 . \beta_1)$$

$$|v_s^{(2)}|_{\max} = \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\, L_2 S_D(\omega_2, \beta_2)$$

$$\begin{aligned} |v_s^{(3)}|_{\max} = {} & \frac{1}{\gamma}[1-(1-\gamma)\varepsilon]\,\lambda_2 L_1 \frac{1-\gamma}{\omega_s^2} \\ & \times \Big\{[1+(1-2\gamma\varepsilon)]^2[\omega_1^2 S_D^2(\omega_1,\beta_1)+\omega_2^2 S_D^2(\omega_2,\beta_2)] \\ & + \beta_1^2\omega_1^2 S_D^2(\omega_1,\beta_1)+\beta_2^2\omega_2^2 S_D^2(\omega_2,\beta_2)\Big\}^{1/2} \end{aligned} \tag{30}$$

All design codes for seismically isolated structures are based on constant velocity spectra, so that the various terms in the above can be related through

$$S_V(\omega, \beta) = S_V H(\beta)$$

where S_V is constant and $H(\beta)$ is a suitable damping modification function that decreases with increasing β and is unity at $\beta = 0{\cdot}05$. Many such functions have been used, either as tables in code documents or as continuous functions. A particularly simple form is the Kawashima–Aizawa function[13]

$$H(\beta) = \frac{1{\cdot}5}{1+40\beta} + 0{\cdot}5 \tag{31}$$

where $H(0) = 2$, $H(0{\cdot}5) = 1$ and $H \to 0{\cdot}5$ and $\beta \to \infty$. Using a constant velocity spectrum of this form and the results for the modal quantities L_1, L_2, ω_1, ω_2, etc. from the earlier section of the paper, (after considerable manipulation) we obtain the following results:

$$\begin{aligned} |v_s^{(1)}|_{\max} &= \varepsilon \frac{S_V}{\omega_b} H(\beta_1) \\ |v_s^{(2)}|_{\max} &= \varepsilon^{3/2}(1-\gamma)^{1/2}\frac{S_V}{\omega_b} H(\beta_2) \\ |v_s^{(3)}|_{\max} &= 2\varepsilon\beta_b\{[1+2(1-2\gamma)\varepsilon+\beta_1^2]H^2(\beta_1) \\ &\quad + [1+2(1-2\gamma)\varepsilon+\beta_2^2]H^2(\beta_2)\}^{1/2}\frac{S_V}{\omega_b} \end{aligned} \tag{32}$$

Clearly, for small values of β_b, say $\beta \approx 0{\cdot}10$, the first term, $|v_s^{(1)}|_{\max}$, is the dominant term. For all values of β_b the second term, $|v_s^{(2)}|_{\max}$, is always much less than the first term and is neglected. The significance of the third term, $|v_s^{(3)}|_{\max}$, depends on the the value of β_b. For the usual values of β_s, the value of β_2^2 is small compared to unity, so the ratio between them becomes

$$\frac{v_s^{(3)}}{v_s^{(1)}} = R = \frac{2\beta_b\{(1+\beta_1^2)H^2(\beta_1)+H^2(\beta_2)\}^{1/2}}{H(\beta_1)} \tag{33}$$

Now $\beta_1 \approx \beta_b$ and

$$\beta_2 \approx \frac{1}{(1-\gamma)^{1/2}}\,(\beta_s + \gamma\varepsilon^{1/2}\beta_b)$$

Suppose we adopt the Kawashima–Aizawa formula for $H(\beta)$ and take $\varepsilon = 1/25$, $\gamma = 1/2$, and $\beta_s = 0{\cdot}02$, then the ratio of the two terms is 0·33 when $\beta_b = 0{\cdot}10$ and increases to 1·80 when $\beta_b = 0{\cdot}50$. To put this into numbers appropriate for an example project, suppose that the code-mandated displacement at the MCE is 76 cm for 5 per cent damping and a period of 2·5 s. To reduce this to a more acceptable level, suppose that linear viscous dampers are added to bring the damping to around 50 per cent, at which point the code reduction factor is 0·57. The displacement is now acceptable, and in code notation the elastic base shear becomes $F_s = KD$, which before was

$$F_s = \frac{K}{M} D \frac{W}{g} = 0{\cdot}50\mathrm{W}$$

and is now reduced to 0·285W, which, again, seems quite reasonable; however, the viscous force

$$F_v = 2\omega\beta M\dot{D}$$

which is out of phase with F_s, is, for $\beta = 0{\cdot}50$ and $\dot{D} = \omega D$, exactly the same as F_s, and the maximum base shear is

$$\sqrt{2}F_s = 0{\cdot}40W$$

To this must be added the contribution from the coupling terms, which is

$$1{\cdot}80 \times 0{\cdot}57 \times 0{\cdot}5W = 0{\cdot}51W$$

so that the total force to determine the interstorey drift and floor acceleration is $0{\cdot}91W$ if directly added or $0{\cdot}65W$ is added by the SRSS method.

This result implies that the addition of dampers (leading to large values of β_b), while controlling the isolator displacement by reducing v_b, has the counter effect of increasing the interstorey drift and floor accelerations. For a constant velocity design spectrum the accelerations generated by the coupling terms become the dominant term. It is not widely appreciated that in base-isolated structures the higher modes, which carry both the floor accelerations and the interstorey drift, are almost orthogonal to the base shear, so that a low base shear is not a guarantee of an effective isolation system. In this respect the effort to improve the performance of the system by adding damping is a misplaced effort and inevitably self-defeating.

CONCLUSIONS

Recent moderate or large magnitude earthquakes in urban areas have led to the realization that current seismic codes may not be adequate, and the code requirements in many countries have been significantly increased. The codes governing the design of seismically isolated structures have always been more conservative than those for conventional structures, and these codes are now so conservative that the benefit of seismic isolation—that it provides functionality (elastic

response) for large ground motion at an affordable cost—may be jeopardized. In an attempt to control the large code-mandated displacements through damping, isolation system designers are incorporating viscous dampers. The approximate analysis given here shows that additional damping does reduce displacement, but at the expense of increasing floor accelerations and interstorey drifts. It does so by increasing the response in the higher modes, and it is not often realized that the higher modes in a base-isolated structure are orthogonal to the base shear, so that reducing base displacement and base shear does not necessarily lead to reduced floor accelerations. A further consequence of the present codes is that by identifying a MCE level for design of the isolators that is a very large and very rare event, it raises the possibility that in the more probable, lower-level earthquake, the isolation system will be too stiff and so heavily damped that it will not move. The result would be much less isolation than promised. While not an issue of life safety, it is possible that large enough floor accelerations could be generated to damage non-structural elements and equipment, in addition to disturbing occupants.

The solutions to this dilemma of how to control displacements for large input level earthquakes while maintaining good performance for low-to-moderate input level earthquakes are several, but mainly reduce to designing a system that is very stiff at low input shaking, softens with increasing input reaching a minimum at the DBE, and then stiffens again at higher levels of input. With frictional systems such as the FPS, this can be achieved by gradually increasing the curvature of the disc at radii larger than the DBE displacement and increasing the surface roughness. For elastomeric isolators it requires using the increased stiffness and increased damping that is associated with the strain-induced crystallization that occurs in the elastomer at strains around 150 to 200 per cent shear strain (depending on the compound). A very detailed description of the results of a shake table test program of such an isolation system is available in a report by Clark *et al.*[14] Other possibilities are to use a compound seismic isolator, such as that proposed by A. G. Tarics.[15]

In each case the approach is to design an isolation system that provides isolation functionality at the DBE level and displacement control at the MCE level. Of course, functionality at the MCE will not be guaranteed and should not be expected, but only by such a strategy can be potential benefits of seismic isolation be realized in the present code environment.

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